

# ENTRUST: Energy Trading Under Uncertainty in Smart Grid Systems

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## Abstract

In this paper, real-time energy trading in smart grid is modeled as an optimization process under uncertainties of demand and price information — a problem perspective that is divergent from the ones in the existing literature. Energy trading in smart grid is affected by demand uncertainties — intermittent behavior of renewable energy sources, packet loss in the communication network, and fluctuation in customers' demands. Energy trading is also affected by price uncertainty due to the demand uncertainties. In such uncertainty-prone scenario, we propose the algorithm named *ENTRUST* using the principles of *robust game theory* to maximize the payoff values for both sides — customers, and grid. We show the existence of robust-optimization equilibrium for establishing the convergence of the game. Simulation results show that the proposed scheme performs better than the existing ones considered as benchmarks in this study. Utilities for the customers are also maximized in order to promote cost-effective and reliable energy management in the smart grid.

*Keywords:* Uncertainty, Real-time Price, Payoff, Smart Grid, Robust Game Theory, Packet Loss, Communication Networks, Energy Management.

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## 1. Introduction

A smart grid is envisioned to facilitate bidirectional electricity flows in terms of renewable and non-renewable energy services [1]. Therefore, communication networks play an important role in order to offer cost-effective energy services for both sides — customers, and grid. Utility provider estimates the real-time energy demand, depending on the demand information received from the customers. Based on the received information, the utility providers reserve energy from the main grid in advance, in an attempt to impart reliable electric supply in the subsequent time periods [2]. The grid decides about real-time price of electricity to maximize the profit incurred while considering customers' participation. Additionally, the customers schedule their appliances considering the total amount of energy required for the day, referred to as the '*day-ahead energy*' in the existing literature [3], according to the real-time price decided by the utility providers.

### 1.1. Motivation

In the smart grid architecture, the customers fulfill their day-ahead energy requirements using the grid and renewable energy sources (such as solar and wind). Plug-in hybrid electric vehicles can charge and discharge their batteries depending on the available energy and the availability of vehicle to grid (V2G) and grid to vehicle (G2V) infrastructure. Therefore, the day-ahead energy from a customer depends on his/her energy requirements and available renewable energy sources (including vehicular energy). The customers also schedule their appliances according to the expected real-time prices at different time-slots [4]. However, the expected demands to the grid are uncertain due to the following constraints — *intermittent availability of renewable energy sources, packet loss in the communication network, and fluctuation in customers' demands*. Environmental constraints have important impacts on the capacity of renewable energy sources. Therefore, demands from customers having renewable energy sources are uncertain [5]. On the other hand, energy management in smart grid is dependent on the information received through the communication network. To support this information exchange mechanism, smart meters are deployed at the customers' end. However, multiple smart meters also try to communicate to the utility provider at the same time, thereby may induce collision in the smart grid communication network [6]. Hence, due to the presence of packet loss in the smart grid communication networks, the demand information to

the grid is not indicative of the actual energy demand from the customers [7]. To address this problem, Misra et al. [8] proposed a game-theoretic energy management scheme, for use in the presence of packet loss in the smart grid communication networks. They used Bayesian game theory to deal with incompleteness of the received energy demand from the customers. The authors showed that the proposed scheme is capable of establishing cost-effective and reliable energy management in smart grid. However, customers' real-time energy requirement may also be changed from the expected one, and consequently, payoff of the utility provider is minimized. On the other hand, real-time price may also be changed drastically due to the change in real-time energy demand, which, in turn, maximizes the customers' energy consumption cost. Consequently, both the customers and the grid compensate, and, thus, the payoff<sup>1</sup> values are minimized for both the sides. Therefore, an energy demand estimation process needs to be developed for executing under these uncertainty constraints, so that the payoff values for both the players — customers and grid — are maximized.

### 1.2. Contributions

In this paper, we propose a scheme named *ENTRUST* for real-time energy trading under uncertainty in the smart grid. The uncertainty is attributed to the fluctuating real-time demand and price information from the customers, and the grid, respectively. Therefore, *ENTRUST* is staged as a two part energy trading scheme — one for the customers, and another for the grid. We consider the uncertainty constraints — intermittent behavior of renewable energy sources, packet loss in the communication network, and fluctuation in customers' demands. The intermittent behavior of renewable energy sources at customers' end and fluctuation in customers' demand are considered as 'imperfect' information to the grid. On the other hand, packet loss in the smart grid communication network is considered as 'incomplete' information to the grid. For simplicity, we have considered that all the uncertainty issues result in 'imperfect' information to the grid. Consequently, the *Robust* game-theoretic approach is used to deal with all the uncertainty issues, in order to establish cost-effective and reliable energy management in a smart grid. Energy trading between the customers and the grid is modeled as a *robust game*. In such a scenario, customers send their expected

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<sup>1</sup>In this work, "customers' payoff" refers to minimization of energy consumption cost incurred by them, while fulfilling their energy requirement. Therefore, higher payoff value indicates the lower energy consumption cost to the customers and vice-versa.

energy demand to the grid while taking into account the uncertain price information from it. On the other hand, the grid reserves energy for the next time-slot, depending on the uncertain energy demand from the customers. Consequently, using a robust optimization technique, the customers and the grid optimize the day-ahead energy consumption and the real-time price, respectively, in an attempt to maximize their individual payoff. A cost effective and reliable energy management scheme is established, appropriately. In summary, the specific *contributions* in this work are as follows.

- We model real-time energy trading in smart grid as an optimization problem under demand and price uncertainties from customers, and grid, respectively.
- *Robust game theory* is used to maximize payoff values for the customers and the grid. We establish the Equilibrium condition of the proposed model. We also elaborate the necessity of using *robust game theory* in the proposed model.
- We propose a new algorithm, named as *ENTRUST*, for real-time energy exchange between the customers and the grid. *ENTRUST* includes a two-part optimization process — one for the customers, and another for the grid. The algorithm for the customers enable the expectation of the real-time price, whereas, the algorithm for the grid executes the expected real-time demand from the customers.

The rest of the paper is organized as follows. In Section 2, we briefly present the literature review for demand and price estimation based on real-time information. Section 3 describes the system model related to the problem. We formulate the robust game strategy as the solution of the problem in Section 4. The results of performance of the proposed scheme are presented in Section 5. Finally, Section 6 concludes the paper, while suggesting some future extensions of this work.

## 2. Related Work

Several issues related to communication, renewable energy, and customers' preference-based energy management in smart grid are addressed separately in [9, 10, 11, 7, 5, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

Zio et al. [9] discussed different uncertainty issues in smart grid in the aspects of market risks, lack of knowledge, different measurement errors, and so on. They discussed different possibilities to analyze the uncertainty

issues in order to deal with it. However, they did not discuss the uncertainty issues from the energy generation and distribution viewpoint which are the most important components in a smart grid energy management systems.

Jiang et al. [5] proposed a demand response model with uncertain renewable energy sources. The authors jointly optimized supply-demand model using dynamic programming. In such a scenario, two dynamic decisions are evaluated — day-ahead, and real-time. In the day-ahead policy, an initial demand response is modeled throughout a day. On the other hand, in the real-time policy, the modeled demand response is changed dynamically according to the real-time situations. However, the authors only considered the uncertainty in the supply of renewable energy (such as solar and wind). The impact of packet loss in the smart grid communication network is studied in [7] in two different aspects — energy demand estimation and associated cost. In such a scenario, the authors showed that with an increase in the packet loss in the communication network, the energy cost to the grid and the customers increases almost exponentially. To counter this problem, they used a queuing model to measure the amount of packet loss at the data aggregator units. According to the measured amount of packet loss, grid estimates the demand from the customers and also calculates the energy cost.

In [10], the author proposed a distributed generation (DG) impact assessment tool for taking into account uncertainty issues related to different renewable energy sources. The proposed assessment tool combines two models — probabilistic and possibilistic. The probabilistic scenario is applied to model some of the cases in a DG environment. On the other hand, the possibilistic scenario is applied to describe rest of the cases where probabilistic model is not applicable. However, similar to other existing works, the proposed model considered only the renewable energy sources as the uncertainty component. Similarly, Saber et al. [12] proposed a resource scheduling scheme under uncertainty caused due to the presence of renewable energy sources and plug-in electric vehicles. They showed that the real-time energy demand from the customers differs from the requested one as unit commitment<sup>2</sup> in the presence of renewable energy sources. Additionally, they considered the issues related to controlling electric vehicles in

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<sup>2</sup>The unit commitment in smart grid is the forecasted energy demand from the customers to the grid in a time period to be consumed in next time period. According to the forecasted energy demand, the grid informs the generated units about the amount of energy to be generated in next time period in order to provide reliable energy service to the customers.

a smart grid system. They used particle swarm optimization (PSO) method to deal with the uncertainty issue. Soroudi et al. [13] discussed different decision making strategies under the uncertainty-prone scenarios in energy management systems as different solutions. They discussed different mathematical tools (such as Monte-Carlo simulation, point estimation, robust optimization, combination of probabilistic and possibilistic methods, and information gap theory) which are useful to address the uncertainty issues in the energy systems. We limit our discussions on these specific mathematical tools in details, while describing the *use of robust optimization method* to deal with the smart grid uncertainties in Section 3.4.

Chen et al. [15] proposed a cost-optimization scheme with renewable energy sources while allowing different levels of delay tolerance for appliances. The delay tolerant appliances are used in off-peak hours. The renewable energy is stored in batteries, and is used during peak-hours, in order to partially offset customers' energy cost. Samadi et al. [16] proposed an energy consumption scheduling scheme under demand uncertainty due to the imperfect knowledge of customers' energy requirements. An optimization model is formed to minimize the energy consumption cost to the customers, while taking into account the imperfect knowledge of the energy demands from the customers.

Table 1: Comparison of Existing Works related to Uncertainty in Smart Grid Systems

(A: Intermittent renewable energy, B: Imperfect information, C: Fluctuations in customers' demand)

Literature	Energy Management	Scheduling	Demand Uncertainty		
			A	B	C
[14], [15], [18]	✓	✓	×	×	×
[5], [10]	✓	×	✓	×	×
[7]	✓	×	×	✓	×
[12], [16]	×	✓	✓	×	×

Table 1 shows the comparison of existing works in different aspects — energy management, energy consumption scheduling, and demand uncertainty. The *analysis* of the existing literature reveals that cost-effective energy management schemes are addressed while considering the uncertainty issue in the smart grid systems only related to renewable energy sources. However, uncertainty of energy demands due to the constraints discussed earlier makes it difficult to estimate the actual real-time energy demands for cost-effective

energy supply to the end-users. In order to address this research lacuna, we propose real-time demand and price estimation schemes under demand and price uncertainty to maximize payoff for both sides (such as customers and grid) due to the changes in *customers' energy requirements, intermittent behavior of renewable energy sources, and packet loss in the smart grid communication networks.*

### 3. System Model

Figure 1 shows the conceptual view of the smart grid architecture, where each customer consumes energy from the grid. All the customers are connected to the data aggregator unit (DAU) for bidirectional information flow. Each of the  $N$  customers schedule their appliances in different time slots, each of which, in turn, is divided into  $T$  time-slots. As an example, the whole day may be divided into 24 equal time-slots, each with one hour duration. Let  $\mathcal{T}$  be a one dimensional vector of different time-slots, so that  $\mathcal{T} = \{1, 2, \dots, T\}$ . Additionally, let there be  $N$  number of customers, and the set of customers is represented as  $\mathcal{N} = \{1, 2, \dots, N\}$ . Each customer has a renewable energy resource,  $E_{r,i,t}$ , and the expected required energy at any time-slot is  $x_{i,t}$ , where  $i \in \mathcal{N}$  and  $t \in \mathcal{T}$ . The DAU sends the total demand at a time-slot  $t \in \mathcal{T}$  to the meter data management system (MDMS) to estimate the total demand to the grid ( $X_t$ ) from all customers at the time-slot  $t$ . As each customer has renewable energy sources, his/her actual demand to the grid is  $x_{i,t}^* = \{x_{i,t} - E_{r,i,t}\}$ , if renewable energy is used at that time-slot. Otherwise, it is the same as the required energy,  $x_{i,t}$ , at the same time-slot,  $t$ . Therefore, each customer sends the estimated energy demand for a time-slot,  $t$ , to the grid in the previous time-slot through the communication link, as shown in Figure 1.

#### 3.1. The Issues of Uncertainty

In smart grid, customers schedule their day-ahead appliances to minimize electricity cost according to certain assumptions, such as on-peak hours or off-peak hours, and available renewable energy. The grid also announces the expected price,  $P_t$ , in different time-slots ( $t$ ). However, the customers' consumption of energy,  $\tilde{x}_{i,t}^*$ , in real-time may not be equate with the estimated one,  $x_{i,t}^*$ , as shown in Figure 1. The real-time demand,  $\tilde{x}_{i,t}^*$  and price,  $P_{i,t}^*$ , are uncertain to the grid and customers, respectively, due to following reasons.

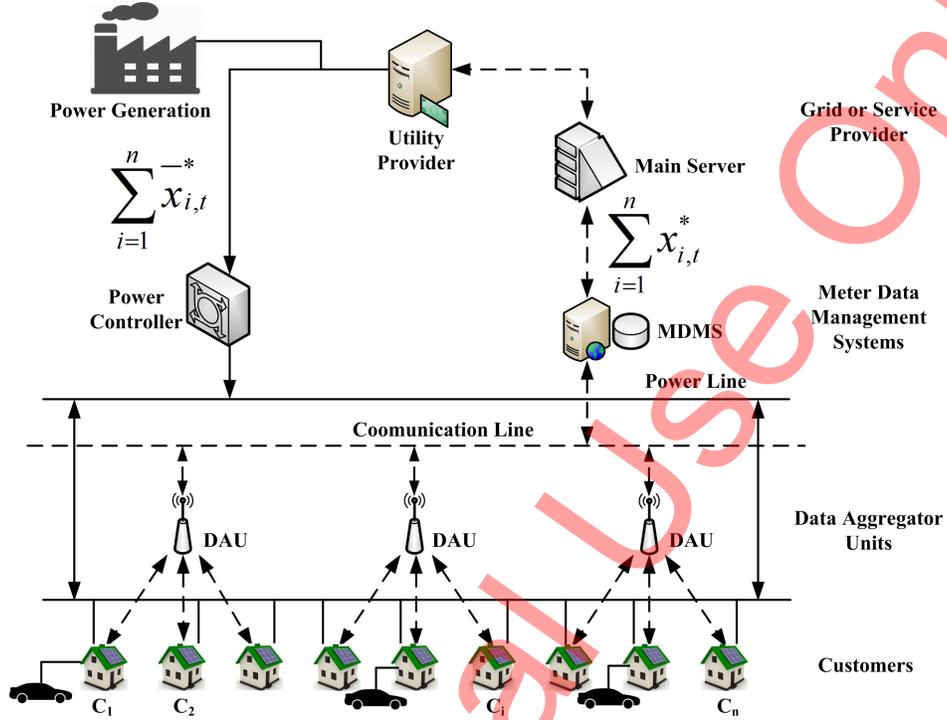


Figure 1: Schematic view of the smart grid architecture

### 3.1.1. Renewable Energy

In the smart grid architecture, each customer is expected to have renewable energy sources. The micro-grids<sup>3</sup> distribute electricity to the end-users with the help of renewable and non-renewable (from main grid) energy sources. In such a scenario, customers' predictions depend on real-time supply from the renewable energy sources. Due to the intermittent behavior of these renewable energy sources, the expected demand and the real-time price may be changed in different time-slots. Therefore, while considering renewable energy sources, uncertainty of these resources, which we term as *unintentional uncertainty*<sup>4</sup>, needs to be taken into consideration. Mathe-

<sup>3</sup>A micro-grid is a small-scale power grid which provides electricity to the customers as the combination of renewable and non-renewable energy sources.

<sup>4</sup>The customers cannot modify the generated energy from renewable energy sources. It depends on the natural resources such as solar and wind power. Therefore, we term the demand uncertainty caused by renewable energy sources as 'unintentional uncertainty'. On the other hand, when the demand uncertainty caused by customers or grid, it can be

matically,

$$\sum_{t=1}^T \tilde{E}_{r,i,t}^* = \sum_{t=1}^T \mathcal{F}(E_{r,i,t}), \forall i \in \mathcal{N} \quad (1)$$

Equation (1) indicates that the real-time energy supply from renewable energy sources differ from the expected one. Therefore, we present the real-time energy supply from the renewable energy sources as a function of the expected one. It is noteworthy that the function is used for generic purpose. It does not follow a specific pattern due to the uncertainty issues (such as speed of wind for wind power and strength of sunlight for solar power) related to the renewable energy sources. This type of uncertainty results in ‘imperfect’ information to the grid, as the grid does not have any information about the behavior of renewable energy sources at the customers’ end.

### 3.1.2. Packet Loss in Communication

In the presence of packet loss in the communication network, expected demand to the grid is always lower than the actual demand from the customers<sup>5</sup>. In such a scenario, payoff of the grid is almost exponentially decreased, as illustrated by the authors in [7]. According to the authors, the received demand to the grid is represented as follows:

$$\sum_{i=1}^N \sum_{t=1}^T x_{i,t}^* = \sum_{i=1}^N \sum_{t=1}^T x_{i,t} (1 - \mathcal{L}_n) \quad (2)$$

where  $\mathcal{L}_n = (1 - (1 - \mathcal{L}^{te})(1 - \mathcal{L}^{cg})(1 - \mathcal{L}^d))$  is the ratio of packet loss in the communication networks, and  $\mathcal{L}^{te}$ ,  $\mathcal{L}^{cg}$ , and  $\mathcal{L}^d$  are the packet losses due to transmission error, congestion, and communication delay, respectively. Packet loss depends on the allowed time-to-live (TTL) of each packet. It is also noteworthy that several re-transmissions may not be useful due to the requirements of demand information from the customers within a specified time period. Therefore, we consider the packet loss rate as an important element of uncertainty. Unlike uncertainty in renewable energy sources, packet loss results in ‘incomplete’ information to the grid, as the grid knows the strategies of the customers. Consequently, the grid can estimate the behavior of packet loss in the communication networks, as only few packets are lost.

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termed as ‘intentional uncertainty’.

<sup>5</sup>We consider that the total energy demand to the grid from the customers is always greater than or equal to zero.

### 3.1.3. Fluctuation in Customers' Demand

Customers may change their expected demand for a particular time-slot in real-time energy consumption at that slot due to the change in their energy requirements. Therefore, real-time demand to the grid can be expressed as:

$$\sum_{i=1}^N \tilde{x}_{i,t}^* = \sum_{i=1}^N \mathcal{F}(x_{i,t}), \forall t \in \mathcal{T} \quad (3)$$

Equation (3) denotes that the real-time energy demand from a customer is a function of the forecasted energy demand. Similar to Equation (1), the variation in customers' demand depends on several factors (such as changes in energy requirements and sources of self-generated energy sources in real-time), which are probabilistic, rather than deterministic. We consider the uncertainty caused by the fluctuations in customers' energy demand as 'imperfect' information, as the grid does not have any information about the changes of energy demand at the customers' end in real-time.

### 3.2. Energy Supply-Demand Model

In general, grid receives energy demands from customers through the DAUs at the distribution side. Upon receiving demand from all the customers, the grid takes decision about the energy to be reserved for fulfilling the energy requirements of the customers. However, due to the presence of uncertainty of energy demand from the customers as presented in Equations (1), (2), and (3), the grid needs to estimate real-time energy demand to maximize its own utility<sup>6</sup>. Therefore, the optimization policy adopted by the grid is expressed mathematically as follows.

$$\text{Maximize } \sum_{t=1}^T \sum_{i=1}^N P_{i,t}^* x_{i,t}^* - \sum_{t=1}^T C_{g,t} \sum_{i=1}^N \tilde{x}_{i,t}^* \quad (4)$$

subject to

$$x_{i,t}^*|_{min} \leq \tilde{x}_{i,t}^* \leq x_{i,t}^*|_{max} \quad (5)$$

$$P_t^{min} \leq P_{i,t}^* \leq P_t^{max} \quad (6)$$

where  $C_{g,t}$  is the cost for unit energy production to the grid. Equation (5) ensures that real-time energy demand,  $\tilde{x}_{i,t}^*$ , always lies in the interval

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<sup>6</sup>The utility of the grid is the difference between income and cost incurred by the grid in order to provide energy to the customers.

$[x_{i,t}^*|_{min}, x_{i,t}^*|_{max}]$ , where  $x_{i,t}^*|_{min}$  and  $x_{i,t}^*|_{max}$  are the minimum and maximum energy demands, respectively. Equation (6) denotes that real-time price,  $P_{i,t}^*$ , is also bounded by the minimum,  $P_{i,t}^{min}$ , and the maximum,  $P_{i,t}^{max}$ , values, while considering customers' participation,  $\forall i \in \mathcal{N}$ .

On the other hand, the objective of the customers is to minimize their energy consumption cost, while fulfilling their energy requirements. Therefore, the customers also optimize their energy consumption cost based on real-time price of energy decided by the grid. Mathematically,

$$\text{Minimize } \sum_{t=1}^T \sum_{i=1}^N \tilde{P}_{i,t}^* \tilde{x}_{i,t}^* \quad (7)$$

subject to

$$x_{i,t}^{min} \leq \tilde{x}_{i,t}^* \leq x_{i,t}^{max} \quad (8)$$

$$P_t^{min} \leq \tilde{P}_{i,t}^* \leq P_t^{max} \quad (9)$$

where  $\tilde{P}_t^*$  is the modified price of energy decided by the grid.

Equations (4) and (7) can be combined as one optimization problem as follows.

$$\text{Maximize } \sum_{t=1}^T \sum_{i=1}^N P_{i,t}^* x_{i,t}^* - \sum_{t=1}^T C_{g,t} \sum_{i=1}^N \tilde{x}_{i,t}^* - \sum_{t=1}^T \sum_{i=1}^N \tilde{P}_{i,t}^* \tilde{x}_{i,t}^*$$

subject to

$$\sum_{i=1}^N \tilde{x}_{i,t}^* \geq 0 \quad (10)$$

$$x_{i,t}^*|_{min} \leq \tilde{x}_{i,t}^* \leq x_{i,t}^*|_{max} \quad (11)$$

$$P_t^{min} \leq P_{i,t}^* \leq P_t^{max} \quad (12)$$

$$P_t^{min} \leq \tilde{P}_{i,t}^* \leq P_t^{max} \quad (13)$$

where  $\mathcal{U}$  denotes the set of uncertain parameters. Equation (10) confirms that the total demand to the grid at any time-slot is always positive.

### 3.3. Real-time Pricing Based on Information

Due to the change in real-time demand,  $\tilde{x}_{i,t}^*$ , from the customer  $i \in \mathcal{N}$ , real-time price,  $P_{i,t}^*$ , is also changed by the grid. For simplicity, we adopt the real-time pricing model proposed by Liang et al. [24]. Real-time price

per unit energy can be modeled as:

$$P_{i,t}^* = \alpha \tilde{x}_{i,t}^{*2} + \beta \tilde{x}_{i,t}^* + \gamma, \forall i \in \mathcal{N}, \text{ and } t \in \mathcal{T}, \quad (14)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are predefined constants. From Equation (14), we see that the customers pay according to their energy consumption. Light weight customers, who require less energy, do not suffer due to the heavy weight customers, who require more energy. Thus, a fair pricing policy is obtained.

### 3.4. The Use of Robust Game Theory

In a smart grid, customers forecast their demand information to the grid in advance, which is expected to be consumed in the next time periods. According to the received demand information, the grid optimizes the energy supply, which is a combination of renewable and non-renewable energy sources. Additionally, the grid also forecasts the expected price of energy to the customers. However, as mentioned in Section 3.1, the customers and grid may not have adequate information due to the different uncertainty issues — intermittent renewable energy, packet loss in communication networks, and changes in real-time demand from customers. Consequently, we have ‘imperfect information’ due to the intermittent behavior of renewable energy sources and changes in customers’ demand. On the other hand, we consider the packet loss in the communication networks as ‘incomplete information’, as both the grid and the customers are not involved in this case. We convert the ‘incomplete information’ to the ‘imperfect’ one to get complete information about payoff values for each strategy [25]. For simplicity, in this paper, we limit our discussion on conversion of ‘incomplete’ to ‘imperfect’. Finally, ‘imperfect information’ is considered for all uncertainty issues to optimize energy trading in the smart grid.

In the proposed scheme, *ENTRUST*, multiple customers are serviced by single service provider (grid). Therefore, we use game theory, an optimization model, which can optimize the payoff values for the users while multiple players are considered. In a smart grid system, there can be different uncertainty issues which need to be considered to provide reliable and cost-effective energy service to the customers. There are few optimization tools such as *information gap decision theory*, *stochastic models*, *fuzzy tools*, and *robust optimization method* which can be used to undertake the uncertainty issues in a system. Therefore, we can use the above mentioned optimization tools from different problem perspectives. For example, using the *information gap decision theory*, we can undertake the issues of imperfect information caused due to packet loss in the communication network in

the smart grid.

In the proposed scheme, the real-time demand and price information from the customers are uncertain to the grid and customers, respectively, due to the uncertainty issues — intermittent behavior of renewable energy sources, packet loss in the communication networks, and changes in customers' demand, as discussed in Section 3.1. Therefore, we need such an optimization tool, which can consider all these uncertainty issues in a smart grid system in a unified manner. Additionally, we also need to have an optimal decision to be executed by both players (customers and grid) to maximize their payoff values. Therefore, we use *robust game theory*, which is capable of addressing uncertainties of this nature, and also has an equilibrium point to evaluate optimal decision [26]. Using *robust game theory*, the customers take optimal decisions for energy demand under price uncertainty, and the grid optimizes the expected real-time demand from the customers considering the worst case scenario of the real-time price and demand information, respectively.

## 4. Robust Demand and Price Estimation

### 4.1. Game Formulation

As discussed previously, we use *robust game theory*-based optimization approach [26] in order to model energy trading under uncertainty. In such an optimization model, customers and grid act as players of the game. We assume that both the players know only a set of possible values of the uncertain payoff function parameters, and represented as  $\mathcal{U} = \{\tilde{x}_{i,t}^*, P_{i,t}^*\}$ . Therefore, both the players try to maximize their payoff values while considering the worst case scenario of the uncertain parameters. Let  $\mathcal{M}$  be the set of players, where  $\{1, 2, \dots, M\} \in \mathcal{M}$ , and let a player  $i \in \mathcal{M}$  have  $\mathcal{A}_i > 1$  possible actions.

**Definition 1.** *The proposed game model is said to be **finite**, if the number of players (grid and customers)  $\mathcal{M}$  and actions  $\mathcal{A}_i$  available to each player  $i \in \{1, 2, \dots, M\}$  are finite.*

In the proposed model, we consider the following parameters:

- *Number of players:*  $\mathcal{M}$ , where  $\mathcal{M} \in \{\mathcal{C}, \mathcal{G}\}$ , where  $\mathcal{C}$  denotes the customers, and  $\mathcal{G}$  denotes the grid.
- *Uncertainty set:*  $\mathcal{U}$ , where  $\{(\tilde{x}_{1,t}^*, \tilde{x}_{2,t}^*, \dots, \tilde{x}_{\mathcal{N},t}^*), (P_{1,t}^*, P_{2,t}^*, \dots, P_{\mathcal{N},t}^*)\} \in \mathcal{U}$ .

- *Payoff uncertainty set*:  $\mathcal{P}$ .
- *Actions taken by the players*:  $\mathcal{A}$ , where  $i \in \mathcal{M}$  and  $|\mathcal{A}_i| > 1$ , and each player's action is denoted as  $a_i, \forall i \in \mathcal{M}$ . Thus,  $\{a_1, a_2, \dots, a_M\} \in \mathcal{A}$ .
- *Strategy of the players*:  $\mathcal{S}$ , where  $\{s^1, s^2, \dots, s^M\} \in \mathcal{S}$ , and  $s^i$  is the individual strategy of each player,  $\forall i \in \mathcal{M}$ .

Therefore,  $\Psi(\mathcal{P}; s^1, s^2, \dots, s^M)$  denotes the expected payoff of player  $i$ , when he/she plays a mixed strategy  $s^i \in \mathcal{S}_{\mathcal{A}_i}$  game. Mathematically,

$$\Psi_i(\mathcal{P}^*; s^1, s^2, \dots, s^M) = \sum_{j_1=1}^{a_1} \cdots \sum_{j_i=1}^{a_i} \cdots \sum_{j_M=1}^{a_M} \mathcal{P}_{(j_1, \dots, j_M)}^{*i} \prod_{i=1}^M s_{j_i}^i \quad (15)$$

As the proposed model is based on incomplete information without private information, the payoff uncertainty set,  $\mathcal{P}$ , is subject to uncertainty. For the worst case scenario, we take the infimum of the payoff uncertainty set,  $\mathcal{P}$ . Therefore, the payoff value can be calculated as follows:

$$\mathcal{P}^*(i) = \arg \max_{\mathcal{U}^i \in \mathcal{S}_{\mathcal{A}_i}} \left[ \inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*, s^{-i}, \mathcal{U}^i) \right], \forall i \in \mathcal{M} \quad (16)$$

In the game model, customers and grid select their actions,  $a_i \in \mathcal{A}$ , simultaneously. Let the customers take expectations under price uncertainty. Thus, the customer incurs cost  $x_{i,t}^*(P_{i,t}^* - P_{i,t}) = x_{i,t}^* \Delta \tilde{C}_p$ , and the grid gains  $x_{i,t}^*(P_{i,t}^* - C_{g,t}) = x_{i,t}^* \Delta \tilde{G}_p$ . On the other hand, when grid takes expectations under demand uncertainty, it suffers with cost  $(x_{i,t}^* - \tilde{x}_{i,t}^*) C_{g,t} = \Delta \tilde{W} C_{g,t}$ , and the customers save an amount of  $(x_{i,t}^* - \tilde{x}_{i,t}^*) P_{i,t}^* = \Delta \mathcal{H} P_{i,t}^*$ . The payoff uncertainty set,  $\mathcal{P}^*$ , can be represented as follows:

$$\mathcal{P}^* = \left( \begin{array}{cc} (0, -\Delta \tilde{W} C_{g,t}) & (\Delta \mathcal{H} P_{i,t}^*, -\Delta \mathcal{H} P_{i,t}^*) \\ (\Delta \mathcal{H} P_{i,t}^* - x_{i,t}^* \Delta \tilde{C}_p, x_{i,t}^* \Delta \tilde{G}_p - \Delta \mathcal{H} P_{i,t}^* - \Delta \tilde{W} C_{g,t}) & (\Delta \mathcal{H} P_{i,t}^* - x_{i,t}^* \Delta \tilde{C}_p, x_{i,t}^* \Delta \tilde{G}_p - \Delta \mathcal{H} P_{i,t}^*) \end{array} \right) \quad (17)$$

where

$$\left( \Delta \tilde{C}_p, \Delta \tilde{G}_p, \Delta \tilde{W} \right) \in \left\{ \left( \lfloor \Delta \tilde{C}_p \rfloor, \lceil \Delta \tilde{C}_p \rceil \right) \times \left( \lfloor \Delta \tilde{G}_p \rfloor, \lceil \Delta \tilde{G}_p \rceil \right) \times \left( \lfloor \Delta \tilde{W} \rfloor, \lceil \Delta \tilde{W} \rceil \right) \right\}$$

#### 4.1.1. Payoff functions for customers

The payoff function  $\Psi_i(\cdot)$  of any customer  $i \in \mathcal{N}$  decreases with an increase in the change of real-time energy demand,  $\Delta \tilde{W}$ , to the grid, and the

real-time price,  $\Delta\tilde{C}_p$ , from the grid. Also,  $\Psi_i(\cdot)$  increases with the increase in the renewable energy,  $E_{r,i,t}$ , of the customer. Mathematically,

$$\frac{\partial\Psi_i(\Delta\tilde{W}, E_{r,i,t}, \Delta\tilde{C}_p)}{\partial\Delta\tilde{W}} < 0, \quad \forall t \in \mathcal{T}. \quad (18)$$

$$\frac{\partial\Psi_i(\tilde{x}_{i,t}^*, E_{r,i,t}, \Delta\tilde{C}_p)}{\partial E_{r,i,t}} > 0, \quad \forall t \in \mathcal{T}. \quad (19)$$

$$\frac{\partial\Psi_i(\tilde{x}_{i,t}^*, E_{r,i,t}, \Delta\tilde{C}_p)}{\partial\Delta\tilde{C}_p} < 0, \quad \forall t \in \mathcal{T}. \quad (20)$$

Equation (18) denotes that the payoff value of the customers decreases with an increase in the real-time energy demand, while keeping the other parameters constant. In a practical scenario, the customers forecast their expected energy demand for the next time period to the grid in advance. According to the forecasted energy demand, the grid optimizes the balance between energy supply and demand from the customers. Further, the grid also forecasts the expected price of energy. However, due to the changes in the actual energy demand from the forecasted one from the customers, the grid needs to buy the extra energy, while the clearing market, price which is higher than the usual one. Consequently, the grid charges higher price than the forecasted one to the customers. As a result, the customers' payoff value decreases with an increase in the real-time energy demand. On the other hand, Equation (19) indicates that the payoff value increases with an increase in the energy supply from the renewable energy sources. Finally, Equation (20) denotes that the payoff value of the customers decreases with an increase in the real-time energy price, while the other parameters are constant. As the energy supply from the renewable energy sources increases, the customers need to buy less energy from the grid. Consequently, the payoff value of the customers increases with an increase in the renewable energy supply.

#### 4.1.2. Payoff function for grid

The payoff function  $\Psi_j(\cdot)$  of the grid  $j \in \mathcal{G}$  increases with the increase in the change in real-time energy demand,  $\Delta\tilde{W}$ , and the effective price,  $\Delta\tilde{G}_p$ , which is computed as the difference between the real-time price,  $P_{i,t}^*$ , and the generation cost,  $C_{g,t}$ . Mathematically,

$$\frac{\partial\Psi_j(\Delta\tilde{W}, \Delta\tilde{G}_p)}{\Delta\tilde{W}} > 0, \quad \forall t \in \mathcal{T}, i \in \mathcal{N}. \quad (21)$$

$$\frac{\partial \Psi_j(\Delta \tilde{W}, \Delta \tilde{G}_p)}{\partial \Delta \tilde{G}_p} > 0, \quad \forall t \in \mathcal{T}, i \in \mathcal{N}. \quad (22)$$

Equations (21) and (22) indicate that the payoff of the grid increases with an increase in the real-time energy demand from the customers and the effective energy price, respectively. Due to an increase in the effective price, i.e., the real-time energy price is higher than the forecasted one, the marginal benefit to the grid also increases. Therefore, the payoff value is also maximized with an increase in the real-time demand from the customers and real-time energy price.

In such a scenario, we evaluate the payoff uncertainty set,  $\mathcal{P}^*$ , for the players as shown in Equation (17). The first and second elements in the tuple represent the customers' and the grid's payoffs, respectively.

**Definition 2.** *The worst case expected payoff is greater than or equal to the expected worst case payoff with the uncertainty set,  $\mathcal{P}^*$ . Mathematically,*

$$\inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, \mathcal{U}^i) \geq \Psi_i\left(\inf_{\mathcal{P}^* \in \mathcal{U}} [\mathcal{P}^*]; s^{-i}, \mathcal{U}^i\right)$$

#### 4.2. Robust Optimization Equilibrium

We now evaluate the existence of equilibrium in the proposed optimization model. According to the formal definition of equilibrium, the following condition holds:

$$\inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*, s^{-i}, s^i) \geq \inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*, s^{-i}, \mathcal{U}^i) \quad (23)$$

Equation (23) presents that the worst case payoff uncertainty of a player  $i$  with strategy  $s^i$  is greater than the payoff uncertainty with the uncertain strategy set,  $\mathcal{U}^i$  for the same player, when the other players' strategies,  $s^{-i}$  are given.

**Definition 3.** *The ex-post-equilibrium defines an equilibrium point, in which each player's strategy is the best response to the other players' strategies under all possible realizations of the uncertain data without private information [26]. Mathematically,*

$$s^i \in \inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, \mathcal{U}^i), \quad \forall i \in \mathcal{M}, \text{ and } \mathcal{P}^* \in \mathcal{U}^i \quad (24)$$

**Property 1.** *If the infimum of the uncertainty set  $\mathcal{U}$  exists, then it is unique, where  $\mathcal{P}^* \in \mathcal{U}^i$ , and  $i \in \mathcal{N}$ .*

*Proof.* We consider that  $\mathcal{U} \subseteq \mathbb{R}$  is bounded, and let  $a$  and  $b \in \mathbb{R}$  be two infimums of the set  $\mathcal{U}$ . According to the definition of infimum rule, both  $a$  and  $b$  are the greatest lower bounds of  $\mathcal{U}$ . Therefore, if  $a$  is the greatest lower bound of  $\mathcal{U}$ , then  $b$  is a lower bound of  $\mathcal{U}$ , and  $a \leq b$ . On the other hand, the contradiction is the same for  $b$ , where  $b \leq a$ . Hence, both the contradictions yield  $a = b$ , and there exists only one greatest lower bound in the uncertainty set  $\mathcal{U}$ .  $\square$

**Lemma 1.** *The proposed scheme has an equilibrium with the strategy  $s^i$ , where  $(s^1, s^2, \dots, s^M) \in \mathcal{S}$  under the uncertainty set while preserving privacy of the players (such as personal data).*

*Proof.* Let, if possible,  $(s^1, s^2, \dots, s^M) \in \mathcal{S}$  be not an equilibrium of the proposed model. Suppose  $\exists i \in \{1, 2, \dots, M\}$  and  $\exists \mathcal{U}^i \in \mathcal{S}_{\mathcal{A}_i}$ , such that:

$$\inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, s^i) < \inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, \mathcal{U}^i)$$

From Equation (24), we get,

$$\Psi_i(\mathcal{P}^*; s^{-i}, s^i) \leq \Psi_i(\mathcal{P}^*; s^{-i}, \mathcal{U}^i), \quad \forall \mathcal{P}^* \in \mathcal{U}$$

Consequently, from the definition of the infimum operator,  $\inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, s^i)$  is the greatest lower bound on  $\Psi_i(\mathcal{P}^*; s^{-i}, s^i)$  over  $\mathcal{P}^* \in \mathcal{U}$ . Therefore,  $\forall i \in \{1, 2, \dots, M\}$ , and  $\forall \mathcal{U}^i \in \mathcal{S}_{\mathcal{A}_i}$ ,

$$\inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, s^i) \geq \inf_{\mathcal{P}^* \in \mathcal{U}} \Psi_i(\mathcal{P}^*; s^{-i}, \mathcal{U}^i) \quad (25)$$

From Equation (25), it is obvious that  $(s^1, s^2, \dots, s^M) \in \mathcal{S}$  is an equilibrium of the proposed scheme.  $\square$

#### 4.3. ENTRUST: The Proposed Algorithm

In this section, we describe the procedure for energy exchange between the customers and the grid. The customers evaluate the optimal energy demand to the grid with uncertain price information to maximize their payoff values. On the other hand, the grid also takes an optimal strategy to decide the real-time price for individual customers to maximize its payoff.

##### 4.3.1. Algorithm for Customer

We present the procedure to evaluate optimal energy demand,  $\tilde{x}_{i,t}^*$  for a customer  $i \in \mathcal{N}$  to the grid in Algorithm 1. The customer calculates the

expected price variation,  $\Delta\tilde{C}_p$ , as follows:

$$\Delta\tilde{C}_p = e^*(P_{i,t}^*) = \arg \min_{e^* \in \mathcal{U}} (P_{i,t}^*, e^*), \quad (26)$$

where  $e^*(P_{i,t}^*)$  is the expected price variation in real-time.

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**Algorithm 1:** Algorithm for customer

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**Input:** Required energy,  $x_{i,t}$ , Renewable energy,  $E_{r,i,t}$ , Expected price,  $P_{i,t}$ , where  $P_{i,t}^{min} \leq P_{i,t} \leq P_{i,t}^{max}$ .

**Output:** Real-time energy demand,  $\tilde{x}_{i,t}^*$ , to the grid.

- 1 Calculate the expected price variation,  $\Delta\tilde{C}_p$ , from Equation (26);
  - 2 **while**  $P_{i,t}^* \leq (P_{i,t} + \Delta\tilde{C}_p)$  **do**
  - 3     Calculate the payoff values,  $\Psi_i, \forall i \in \mathcal{N}$ , for all possible realizations of the price uncertainty, where  $P_{i,t}^* \in \mathcal{U}^i$ , from the payoff uncertainty set in Equation (17);
  - 4 Select the optimal demand,  $\tilde{x}_{i,t}^* = \arg \max_{\mathcal{P}^* \in \mathcal{U}^i} \mathcal{P}^*$ , to maximize the payoff;
  - 5 Send optimal energy demand,  $\tilde{x}_{i,t}^*$ , to the grid in real-time  $t$ ;
- 

#### 4.3.2. Algorithm for Grid

Algorithm 2 presents the procedure followed by the grid to optimize real-time price,  $P_{i,t}^*$ , for each customer  $i \in \mathcal{N}$ . We also use the expected load variation for each customer to calculate the expected real-time demand,  $\tilde{x}_{i,t}^*$ , which is represented as:

$$\Delta\tilde{W} = e^*(x_{i,t}^*) = \arg \min_{e^* \in \mathcal{U}} (x_{i,t}^*, x_{-i,t}^* e^*), \quad (27)$$

where  $e^*(x_{i,t}^*)$  is the expected load variation of customer  $i \in \mathcal{N}$ .

## 5. Performance Evaluation

We simulated the proposed scheme in NS-3 (<http://www.nsnam.org>). The simulation parameters are listed in Table 2. We consider the values of the predefined parameters  $a > 0$ ,  $b = 0$ , and  $c = 0$ , in a manner similar to the existing literature [8]. The simulation area is considered as 2 Km  $\times$  2 Km with 50 number of customers. The day-ahead energy requirement of a cus-

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**Algorithm 2:** Algorithm for grid

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**Input:** Received energy demand,  $x_{i,t}^*$ , from customer  $i \in \mathcal{N}$ , where  $x_{i,t}^*|_{min} \leq x_{i,t}^* \leq x_{i,t}^*|_{max}$ , and generation cost,  $C_{g,t}$ , per unit.

**Output:** Real-time price,  $P_{i,t}^*$ , for the customer  $i$ .

- 1 Calculate expected load variation,  $\Delta\tilde{W}$ , from Equation (27);
  - 2 **while**  $\tilde{x}_{i,t}^* \leq (x_{i,t}^* + \Delta\tilde{W})$  **do**
  - 3     Calculate payoff values,  $\Psi_j, \forall j \in \mathcal{G}$ , for all possible realizations of the demand uncertainty, where  $\tilde{x}_{i,t}^* \in \mathcal{U}^i$ , from the payoff uncertainty set in Equation (17);
  - 4     Select the optimal price,  $P_{i,t}^* = \arg \max_{\mathcal{P}^* \in \mathcal{U}^i} \mathcal{P}^*$ , to maximize payoff;
  - 5     Send the optimal price,  $P_{i,t}^*$ , for the customer  $i$  in real-time  $t$ ;
- 

customer is taken as 10–30 kWh<sup>7</sup>. The packet loss rate is considered as 5–20% [7]. Finally, average cost for energy generation to the grid is considered as 5 Cents/kWh<sup>8</sup>. Different performance metrics are considered for evaluating the performance of the proposed scheme — *effect of demand uncertainty, reliability of energy supply, energy cost, and utility* to the customers and grid. It is noteworthy that all the results are obtained in this work by considering all the uncertainty issues — changes in renewable energy supply and customers’ demand, and packet loss in the communication networks. Due to the packet loss in the communication networks, the received information is always less than or equal to that sent. On the other hand, due to the changes in renewable energy sources and customers’ demand, both the real-time supply and demand either increase or decrease. However, all the sources of uncertainty are probabilistic in nature, rather than deterministic. Consequently, we do not explicitly mention the rate of change in packet loss, renewable energy sources and customers’ demand. However, we consider the effects of all the uncertainty issues in the smart grid to obtain the results, as mentioned in Section 4.

We compare the performance of the proposed scheme, ENTRUST, with the existing scheme where only the renewable energy sources are considered as the sources of uncertainties in smart grid systems (such as Jiang et al. [5] and Soroudi [10]). Jiang et al. [5] proposed a demand response scheme

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<sup>7</sup>[http://www.eia.gov/electricity/sales\\_revenue\\_price/xls/table5\\_a.xls](http://www.eia.gov/electricity/sales_revenue_price/xls/table5_a.xls)

<sup>8</sup>OpenEI Transparent Cost Database (<http://en.openei.org/apps/TCDB/>)

Table 2: Simulation Parameters

Parameter	Value
Number of grids	1
Number of customers	50
Simulation area	2 Km $\times$ 2 Km
Demand of a customer	10–30 KWh
Self-generation	2–5 KWh
Packet loss rate	5–20%
Average cost for supply	5 Cents/KWh
Predefined constants [8]	$a > 0, b = 0, c = 0$

in smart grid in the presence of uncertain renewable energy sources. In such a scenario, the intermittent behavior of the renewable energy sources is considered as the source of uncertainty in the smart grid systems. Similarly, Soroudi [10] proposed a possibilistic model for distribution grid (DG) impact assessment in an uncertain environment. In such a model, the renewable energy sources are considered as the uncertainty factors.

However, in the proposed scheme, *ENTRUST*, we consider different types of uncertainties in the smart grid systems — intermittent behavior of renewable energy sources, changes in customers’ demand, and packet loss in the smart grid communication networks, as discussed in Section 3. Consequently, *ENTRUST* addresses these uncertainties to provide reliable and cost-effective energy supply to the customers.

## 5.1. Results and Discussion

### 5.1.1. System Dynamics

Figure 2 presents the system dynamics of obtained energy demands using different schemes at each time period. The grid reserves energy as unit commitment for a subsequent time-slot, depending on the received energy demand from the customers. However, real-time demand may be changed due to different constraints, as discussed in Section 3. Therefore, we show the variations of energy demand from three aspects — received demand (as in [5, 10]), estimated demand (proposed), and real-time demand. In the proposed scheme, we estimate the expected energy demand in real-time from the customers, while considering the uncertainty issues related to intermittent behavior of renewable energy sources, changes in customers’ energy demand and packet loss in the communication network. It is noteworthy that, due to the probabilistic nature of the estimation process, the exact values of

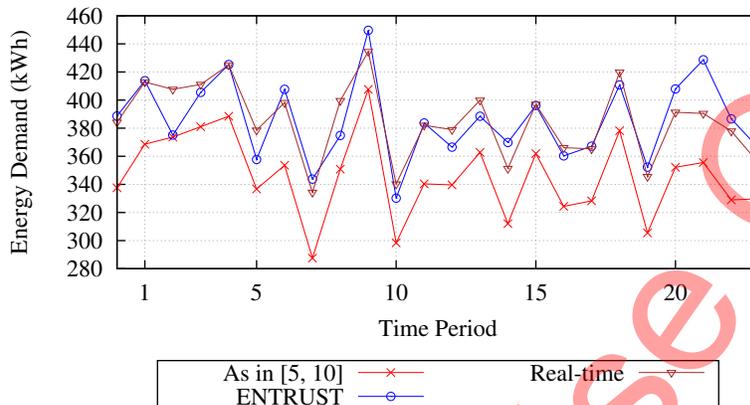


Figure 2: Energy demand at different time periods from customers

the uncertainty parameters are not presented in a deterministic manner. However, we consider all the uncertainty parameters in each time period, as shown in Figure 2. Additionally, the presented energy demand dynamics is used to get the subsequent results.

### 5.1.2. Energy Demand from Customers

Figure 3 shows the cumulative energy demand at different time periods. We see that the proposed scheme, ENTRUST, estimates the energy demand from customers adequately. On the other hand, the estimated energy demand using the existing schemes is lower than the real-time demand, as they do not consider the uncertainty issues related to packet loss and changes in customers' demands. Therefore, in case of the existing schemes, the additionally required energy demand in real-time increases the *peak-to-average* ratio, and moreover, it may cause the grid to fail. However, using ENTRUST, the grid estimates the energy demand from the customers adequately, which, in turn, does not create extra load on the grid. Therefore, the proposed scheme, ENTRUST, is capable of providing adequate energy services to the customers, while considering the uncertainty issues. The peak-to-average ratio is calculated as follows:

$$\alpha_{peak-avg} = \frac{E_{demand} - E_{avg}}{E_{avg}} \quad (28)$$

In the existing schemes (as in [5, 10]), the grid estimates the real-time demand according to the received demand information from the customers

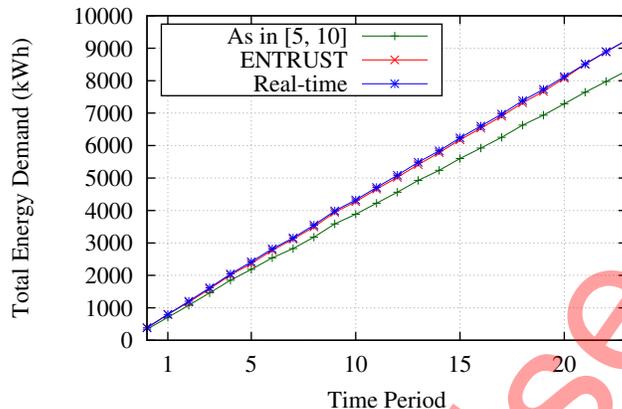


Figure 3: Total aggregated energy demand from customers

without considering all the uncertainties present in the smart grid, as discussed in Section 3. Accordingly, the grid calculates the average energy demand from the customers. However, the difference between the estimated demand and actual demand increases in real-time, as the actual demand is more than the calculated one. Consequently, the peak-to-average ratio increases using the existing schemes. In contrast, as the proposed scheme, ENTRUST, estimates the real-time demand from the customers adequately and it is less fluctuated from the expected one, the difference between the demanded energy and the average energy is less compared to the existing ones. Consequently, peak-to-average ratio is minimized using the proposed scheme, ENTRUST, which is one of the important aspects of the smart grid.

### 5.1.3. Reliability of Energy Supply

As discussed in Section 1, smart grid is envisioned to increase the reliability of energy supply to the customers. The reliability of energy service is calculated as the ratio between the demanded energy from a customer and the supplied energy by the grid to the customers, while incurring the same unit energy consumption cost. We compare the reliability of energy service using the proposed scheme, ENTRUST, with the existing schemes, as in Figure 4. The reliability of energy service to the customers decreases with the existing schemes, as the grid does not receive adequate energy demand information (as shown in Figure 3) from the customers in an uncertain environment. On the other hand, the proposed scheme, ENTRUST, provides more reliable energy services to the customers compared to the existing ones,

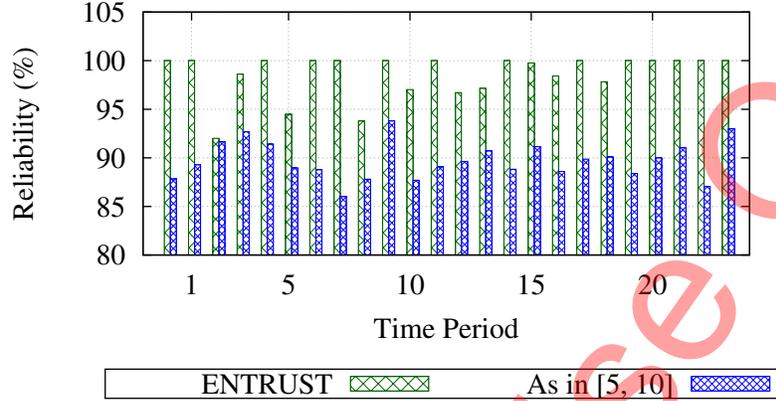


Figure 4: Reliability of energy service to customers

while considering different uncertainty issues, as shown in Figure 4.

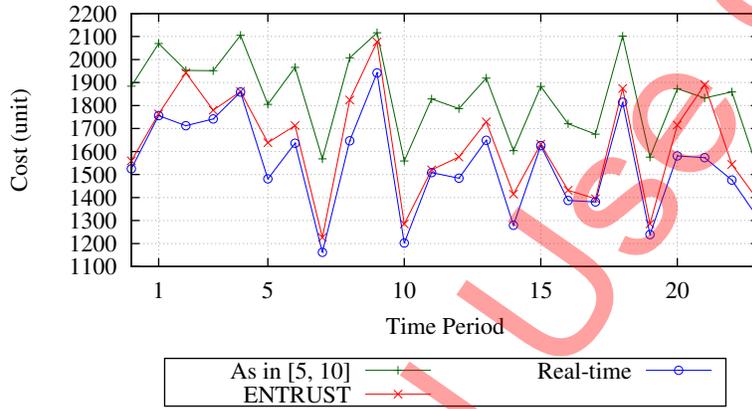
#### 5.1.4. Energy Cost

Due to the uncertainty in energy demand from the customers, as shown in Figure 3, the unit energy consumption price is also uncertain to the customers. According to the expected demand from a customer, the grid decides the real-time price, as depicted in Equation (14). In ENTRUST, we consider the real-time price, as calculated from the estimated energy demand to the grid. We see that ENTRUST estimates the adequate energy consumption cost incurred by the customers, as shown in Figure 5(a).

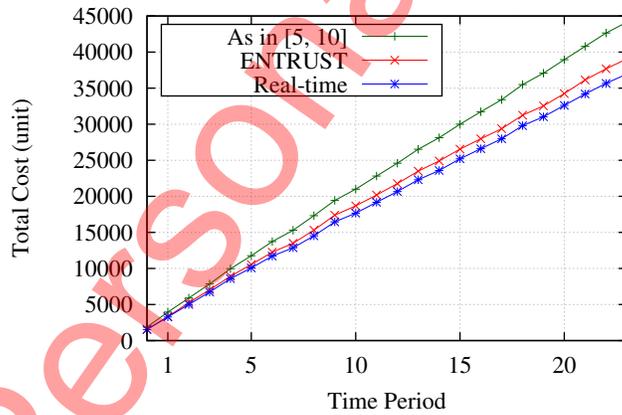
Additionally, Figure 5(b) presents the cumulative energy consumption cost to the customers. We see that the energy cost to the customers is also minimized using the proposed scheme compared to the existing ones, as it estimates the real-time energy demand adequately. On the other hand, in the presence of unit commitment scenario, using the existing schemes, the grid needs to buy extra energy from shared energy markets in real-time, which, in turn, increases the energy consumption cost to the customers. Therefore, we see that the use of ENTRUST yields significant results to promote a cost-effective energy supply to the customers.

#### 5.1.5. Utility

We present the utility of the customers in Figure 6. It is evident that the utility of the customers also increases with the proposed scheme, ENTRUST, over the existing scheme. As ENTRUST estimates the real-time energy demand from customers adequately, the grid reserves the same amount of



(a) Energy cost to the customers



(b) Total aggregated energy cost to the customers

Figure 5: Energy cost to customers

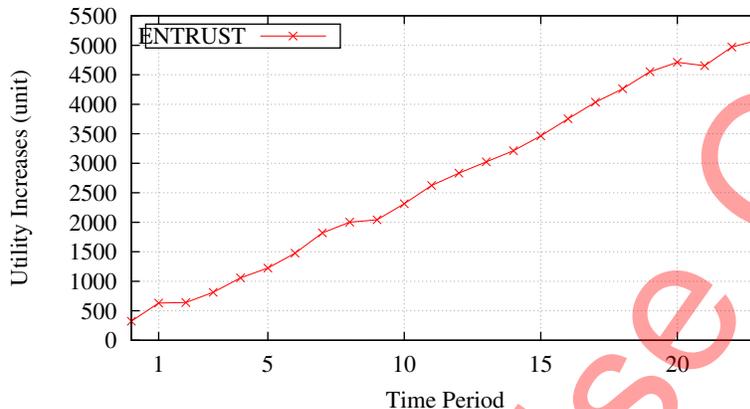


Figure 6: Utility of customers

energy to provide reliable energy services. Therefore, the grid does not need to procure extra energy by paying the market price, which is higher than the normal one. Consequently, the customers incur lower energy cost by using the proposed scheme, which, in turn, maximizes the utility of the customers.

## 6. Conclusion

In this paper, we proposed a scheme for energy management under different uncertainties concerning demand and price in a smart grid. The performance of the algorithms proposed in the existing literature on the issue of energy management, in general, suffers from uncertainty constraints. Therefore, we modeled the energy management scheme as a robust optimization approach using *robust game theory* to account for these uncertainty constraints. In the proposed model, the customers and the grid act as players of the game. The theoretical analysis of equilibrium of the game model is also presented. The simulation results showed that using the proposed approach, improved energy management over the existing ones, is achievable.

The future extension of this work includes improvement in the expectation of the real-time demand from the customers in order to overcome the overestimation issue. We saw that the proposed scheme overestimates energy demand from customers in case of very low packet loss rate. Therefore, in future, we also plan to incorporate this issue in the smart grid systems. It also includes the establishment of a network architecture for smart grid to

minimize packet loss in the communication network. This will enable us to achieve improved reliability and cost-effectiveness in energy management.

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